

# SOME DIMENSIONS OF THE BALANCED- BUDGET MULTIPLIER

J.S. KHOKHAR

16598

The paper is a theoretical attempt to develop six versions of the balanced budget theorem. It highlights the point that it is not easy to know the effectiveness of the balanced budget multiplier unless its context is clearly specified.

## I

### Introduction

At the outset, it is interesting to note what Herbert Stein had recently written: "In the fall of 1971 when the economy was very weak, President Nixon announced a program for cutting government expenditures and taxes equally. He presented it as a policy for stimulating the economy. At once there came a reply from Professor Paul Samuelson of the Massachusetts Institute of Technology, a Nobel laureate, that equal reductions in taxes and expenditures would not stimulate but would depress the economy. This was, of course, an application of the balanced-budget theorem straight out of the elementary textbook. But if Samuelson had been writing a journal article, he would have pointed out that much depended on the nature of both the taxes and the expenditures and that without knowing more he could not predict the outcome."<sup>1</sup>

The students of BA(H) Economics and B.Com(H) are taught the balanced-budget theorem in the context of a Keynesian economy. The theorem states that, if the government spends as much as it collects through tax, the addition to the national income will be equivalent to the tax revenue. It implies that the balanced-budget multiplier, the ratio of change in income to change in government expenditure when the government expenditure is exactly the same as the tax revenue, will always be equal to one. But the students seldom realise that the theorem holds true only under some specific conditions. Investment is exogenous, the budget is initially balanced, only commodity market is operating, and no trade exists between the economy and the rest of the world. Nevertheless, it seems equally important to investigate the effectiveness of the multiplier when any one of the mentioned conditions is altered. To look into that aspect of the multiplier, is the main purpose of this attempt.

---

Dr. J.S. Khokhar is Reader, Deptt. of Economics, Shri Ram College of Commerce, University of Delhi, Delhi.

## II

**The Theorem**

First of all, we shall prove the balanced-budget theorem. For that, the following symbols are required.

Y=National income, Rs.;  
 C=Consumption, Rs.;  
 I=Investment, Rs.;  
 T=Tax, Rs.;  
 G=Government expenditure, Rs.; and  
 Y(d)=Y-T, disposable income, Rs.

The economy shows the following features:

$$C=f[Y(d)] \quad (1)$$

$$=a+bY(d), 0<b<1 \quad (1.1)$$

$$G=T \quad (2)$$

Equation (1.1) represents a linear consumption function, with a being the minimum consumption, and b the marginal propensity to consume (ratio of the change in consumption to the change in income). Obviously, equation (2) indicates that the budget is initially balanced.

We already know that the level of income is determined when the aggregate demand (AD) is equal to the aggregate supply (AS).

$$AD=C+I+G \quad (3)$$

$$=a+b(Y-T)+I+G \quad (4)$$

$$AS=Y \quad (5)$$

Therefore, equating (5) with (4), we can derive

$$Y = [1/(1-b)][a-bT+I+G] \quad (6)$$

From (6), we get

$$(dY/dG; G=T) = 1 \quad (7)$$

$$= [1(1-b)] + [-b/(1-b)] \quad (7.1)$$

$$= (dY/dG) + (dY/dT) \quad (7.2)$$

Equation (7) is the balanced-budget multiplier, which is decomposed into two multipliers, government and tax, as is evident from the relations (7.1) and (7.2). The tax multiplier,  $dY/dT = -b/(1-b)$ , indicates that, when one rupee of additional tax reduces the consumption by the amount b, the national income will fall by  $b/(1-b)$ . ON the other hand, the government multiplier,  $dY/dG = 1/(1-b)$ ,

reveals that, if the government spends its additional revenue of one rupee and AD goes up by the same amount, the national income will increase by  $1/(1-b)$ . Therefore, when both the multipliers are operating together, the balanced-budget theorem will hold good, the extra income being exactly the same as extra government expenditure.

### III

#### Version I

Let us replace I, exogeneous level of investment, by the following form of induced investment.

$$I=I(Y) \quad (8)$$

$$=h+iY \quad (8.1)$$

Where

$h$ =Minimum investment, Rs., and

$i$ =Marginal propensity to invest,  $0 < i < 1$ .

Substituting (8.1) into (6), equilibrium condition for income, we get

$$Y=[1/(1-b-i)][a-bT+h+G] \quad (9)$$

From (9), we can derive

$$(dY/dG, G=T) = (1-b)/(1-b-i) > 1 \quad (10)$$

$$= [1/[1-b-i)] + [-b/(1-b-i)] \quad (10.1)$$

$$= dY/dG + dY/dT \quad (10.2)$$

Both the government and the tax multipliers remain economically viable factors so long as  $b$  and  $i$  are such that  $(1-b-i)$  is strictly between zero and  $b$ . The derivation (10) shows that the balanced-budget theorem breaks down with the introduction of the induced investment function. The comparisons of (10.1) and (10.2) with (7.1) and (7.2) indicate that the induced behaviour of investment has increased the effects of both the government and the tax multipliers. When the government increases its spending by one rupee, AD rises and, as a result, national income rises. This addition to income induces more investment and the national income further rises. The extra income is equal to  $1/(1-b-i)$ , which is greater than  $1/(1-b)$ . On the other hand, when the tax is raised by one rupee, AD falls and, as a consequence, national income comes down. This cut in national income reduces investment and then income. The total reduction in national income is equal to  $b/(1-b-i)$ , which is greater than  $b/(1-b)$ . Therefore, the net increase in income is equivalent to  $(1-b)/(1-b-i)$ , which is greater than one.

IV<sup>th</sup> Q<sup>r</sup> 3 1965  
 11 11 11

### Version 2

Now we shall incorporate the working of money market into the balanced-budget theorem. The money market determines the rate of interest by equating the demand for money with the supply of money. Money is demanded for three purposes: transaction-money needed for daily requirements; precautionary-money wanted for unpredictable events, and speculative-money required to be invested in bonds so that some advantage could be taken of the fluctuations in the market rate of interest. The first and the second demands are positively associated with income; whereas, the third demand, for any given level of income, is negatively linked with the market rate of interest. Therefore, the total demand for money can be written as

$$M_d = M_1(Y) + M_2(R) \quad (11)$$

$$M_1 = wY \quad (11.1)$$

$$M_2 = p - qR \quad (11.2)$$

Where

$M_d$  = Total demand for money, Rs.;

$M_1$  = Money demanded for transaction and precautionary purposes, Rs.;

$M_2$  = Money demanded for speculative purposes, Rs.;

$R$  = Market rate of interest, percentage, and

$w$ ,  $p$  and  $q$  = Parameters,  $0 < w < 1$ .

If  $M_s$  stands for the supply of money, Rs.; the equilibrium condition of the money market, equality between  $M_s$  and  $M_d$ , gives us

$$R = (1/q)[wY + p - M_s] \quad (12)$$

The rate of interest inversely affects the level of investment; so  $I$ , which is fixed for the theorem, can be treated as

$$I = n - kR \quad (13)$$

Where

$n$  and  $k$  = Parameters.

From (13), (12), and (6), we get

$$Y = [1/(1 - b + kw/q)] [a - bT + n - (k/q)(p - M_s) + G] \quad (14)$$

From (14), we can derive.

$$\begin{aligned} (dY/dG, G=T) &= (1-b)/(1-b+(kw/q)) < 1 & (15) \\ &= 1/[1-b+(kw/q)] + [-b/(1-b+(kw/q))] & (15.1) \\ &= dY/dG + dY/dT & (15.2) \end{aligned}$$

The government and the tax multipliers become economically viable propositions only when  $kw/q$  and  $b$  are such that  $(1-b+kw/q)$  strictly lies between zero and  $b$ . The relation (15) reveals that the balanced-budget theorem has collapsed. Comparing (15.1) and (15.2) with (7.1) and (7.2), we find that one rupee of government expenditure raises income by  $1/(1-b+(kw/q))$ , which is smaller than  $1/(1-b)$ . That implies the operation of money market has reduced the impact of the government expenditure on the national income. When the government increases its spending by one rupee, the demand for resources goes up. This phenomenon raises the rate of interest. As the rate of interest increases, the amount of investment falls. The fall in investment leads to a reduction in income. Therefore, the additional income is not so large as what it would be when the money market was not in existence. On the other hand, the functioning of money market has also reduced the influence of tax multiplier on the national income, as is apparent from  $b/(1-b+(kw/q))$  being smaller than  $b/(1-b)$ . When the government collects one rupee through tax, the consumption level falls, resulting in a cut in the demand for resources. The fall in the demand for resources reduces the rate of interest. As the rate of interest falls, the amount of investment rises. The rise in investment leads to an increase in the national income. Therefore, the reduction in national income is not so high as what it would be when the money market was not operating.

## V

## Version 3

The role of foreign sector can be introduced into the the theorem by considering the following import function.

$$\begin{aligned} M &= M(Y) & (16) \\ &= f + mY & (16.1) \end{aligned}$$

Where

$M$ =Imports, Rs.;  
 $f$ =Intercept, minimum imports, Rs.; and  
 $m$ =Marginal propensity to import,  $0 < m < 1$ .

Let us assume that the level of exports is already known and is denoted by  $X$ . With the emergence of foreign trade, the aggregate demand, (3), becomes

$$AD = C + I + G + X - M \quad (17)$$

Equating (17), AD, with (5), AS, we get

$$Y = [1/(1-b+m)][a-bT+I+G-f+x] \quad (18)$$

From (18), we can derive

$$(dY/dG, G=T) = (1-b)/(1-b+m) < 1 \quad (19)$$

$$= [1/(1-b+m)] + [-b/(1-b+m)] \quad (19.1)$$

$$= dY/dG + dY/dT \quad (19.2)$$

It is obvious that both the government and the tax multipliers are economically viable factors as long as  $b$  and  $m$  are such that  $(1-b+m)$  strictly lies between zero and  $b$ . According to (19), the balanced budget theorem breaks down. Comparing (19.1) and (19.2) with (7.1) and (7.2), we find that the effects of both the government and the tax multipliers have fallen. When the government spends one rupee, the national income rises. The additional income increases the level of consumption, but some part of the additional consumption is diverted from the domestic goods to the foreign products. As a result, the addition to AD is not so large as what it would be when there was no access to the imports of consumption goods. As regards the tax multiplier, one rupee of additional tax reduces consumption. Some part of the fall in consumption leads to a cut in the demand for foreign goods. Therefore, the fall in AD is not so large as what it would be when there was no trade.

Relevant to the context is the consideration of a deficit or a surplus in the balance of payments. If the government somehow or other keeps BoP in equilibrium,  $X$  (supply of foreign exchange) =  $M$  (demand for foreign exchange), the balanced-budget theorem will certainly hold good. That is to say, the balanced budget multiplier will be equal to one.

Another consideration relevant to the context is whether export earnings,  $X$ , should be linked to national income. If the economy is capable of exporting more at a higher level of income,  $X$  of (18) should be substituted by the following form of export earnings.

$$X = zY, 0 < z < 1 \quad (20)$$

Therefore, equation (18) can be written as

$$Y = [1/(1-b+m-z)][a-bT+I+G-f] \quad (21)$$

Now, from (21) we can get

$$(dY/dG, G=F) = (1-b)/(1-b+m-z) \quad (22)$$

Comparing (22) with (19), we come to know that the behaviour of exports has increased the magnitude of the balanced-budget multiplier. The reason is

that the export earnings raise the aggregate demand in proportion to the growth of national income.

## VI

## Version 4

Let us assume that the tax, which has so far been exogeneous, is related to the national income in the following manner.

$$T = s + tY \quad (23)$$

Where

$s$  = Minimum tax, Rs.; and

$t$  = Marginal tax,  $0 < t < 1$ .

According to the relation (23), the budget can also be balanced by raising  $s$ , minimum tax, adequately. If the government does so, the  $T$  in (6) shall be replaced by (23), and the equilibrium level of income can be written as

$$Y = [1/(1-b+bt)][a-bs+I+G] \quad (24)$$

$$(dY/dG, ds=dG) = (1-b)/(1-b+bt) < 1 \quad (25)$$

$$= [1/(1-b+bt)] + [-b/(1-b+bt)] \quad (25.1)$$

$$= dY/dG + dY/ds \quad (25.2)$$

The second part of (25.2), which is equivalent to  $-b/(1-b+bt)$ , can be called marginal tax multiplier. It shall be mentioned that both the multipliers are economically justified only if  $b$  and  $t$  are such that  $(1-b+bt)$  strictly remains between zero and  $b$ . According to (25), the balanced-budget theorem breaks down. The comparisons of (25.1) and (25.2) with (7.1) and (7.2) reveal that the financing of the budget by raising the minimum tax has reduced the effects of both the marginal tax and the government multipliers. When the government spends one rupee, AD rises directly. This leads to an increase in income. The additional income raises the tax revenue, and the consumption does not rise to the extent expected in the absence of (23). As a result, the addition to AD is less than what it would be when tax was not related to income. As regards the marginal tax multiplier, we can say one rupee of additional tax will reduce consumption. This cut in AD will decrease income. Because of (23), the disposable income will further fall. As a consequence, the reduction in AD is more than expected in the absence of (23).

But there is a problem. The equilibrium level of income will not be stable if the government is bent on keeping the balance between its revenue and expenditure over the years to come. At the start of the first year, when the level of income is known, the government spends as much as it has collected in accordance with (23). As a result, the level of income rises and the government gets

more revenue to spend for the second year. As the government expenditure goes up, the national income further rises, yielding more revenue to the government. The government spends more for the third year and the national income further rises. This process will not stop unless the government decides to fix the amount of its expenditure for ever.

## VII

## Version 5

Let us now introduce a general consumption function into the balanced-budget theorem. In that case, we shall have to replace (1.1) with (1). As a result, when aggregate demand is equal to aggregate supply, we can write equation (6) as

$$Y = f[Y(d)] + I + G \quad (26)$$

Considering  $Y(d) = Y - T$  and taking the total derivative of (26), we can obtain

$$[dY/dG, G=T] = 1 \quad (27)$$

According to (27), the balanced-budget theorem holds good, implying that the theorem is not affected by any functional form of consumption.

## VIII

## Version 6

The government expenditure,  $G$ , has so far been treated to be a kind of investment, making an addition to AD in the same manner as investment does. Alternatively, it can take the form of a transfer to the public. If so, there would not be any change in AD, because the disposable income, income minus tax plus transfer, remains unchanged. As a result, there will be no impact of the balanced-budget multiplier on the national income. In other words, the multiplier will be zero.

## IX

## Conclusion

The balanced-budget theorem has been developed into a few versions. The versions point out that it is very difficult to know the effectiveness of the balanced-budget multiplier unless its context is specified.

## NOTES &amp; REFERENCES

1. See Stein, H., "What do Economic Advisers Do?", *Spain*, May 1992, p. 27, lines 11-24.
2. Sodersten, Bo., *International Economics*; Macmillan Education Ltd., 1989.



3. Branson, W.H., *Macroeconomic Theory and Policy*, UBS, New Delhi, 1990.
4. Dernburg, T.F. and D.M. McDougall, *Macroeconomics*, McGraw-Hill Kogakusha Ltd., New Delhi, 1976.
5. Haavelmo, T., 'Multiplier Effects of a Balanced Budget', in *Readings in Fiscal Policy*, American Economic Association, George Allen and Unwin Ltd., London, 1955, pp. 335-343.
6. Mukherjee, B. and V. Pandit, *Mathematical Methods for Economic Analysis*, Allied Publishers Pvt. Ltd., New Delhi, 1989. -
7. Samuelson, P.A., *Economics*, McGraw-Hill International Book Co., London, 1980.
8. Shapiro, E., *Macroeconomic Analysis*, Galgotia Publications Pvt. Ltd., New Delhi, 1988.